Geometry, topology and symmetry in strongly correlated materials

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Research Overview

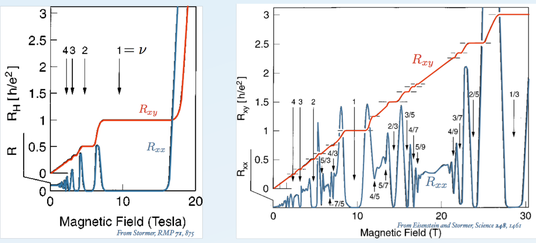
Topological phenomena in strongly correlated systems are poorly understood due to the fact that they lie outside the conventional paradigm of symmetry breaking; nevertheless, they hold great potential in engineering new devices and quantum computation by virtue of their novel electronic properties. To advance our understanding of these systems, I focus on the following areas of research:

1.) Developing a unified theory of the fractional quantum Hall effect and fractional Chern insulators. By incorporating aspects of the quantum geometry of the single particle Hilbert space in these systems, I have developed new analytic and numerical tools which aid the search for more experimentally accessible realizations of quantum Hall phenomena.

2.) The role of crystal symmetries in topological phenomena. Using topological arguments, I've improved constraints on symmetry enforced degeneracies and complete the classification of short-range entangled topological states with crystal symmetries.

3.) The exploration of topological phenomena in periodically driven systems. I plan to characterize these topologically robust phenomena, study the effects of interactions, the physical consequences of the classification, and connect these with experiments such as those on the fractional Josephson effect.

Background



Experimental plots of longitudinal and transverse conductance in the integer quantum Hall effect (left) and fractional quantum Hall effect (right). Plateaus occur at rational multiples of h/e^2.

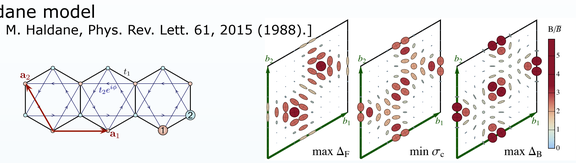
The quantum Hall effect (QHE) describes topologically quantized transport in two-dimensional electron gases in a transverse magnetic field. The theory of the QHE is built largely around the special properties of single-particle states in a magnetic field (Landau levels). This is particularly true of the F(ractional)QHE, where the construction of model wave functions with built-in analyticity forced by a restriction to the lowest Landau level has played an extremely important role in the theoretical development.

A flurry of interest in the field was set off by the recent insight that FQHE-like phases of matter may also arise in topologically nontrivial insulators with partially filled flat bands, or fractional Chern insulators (FCIs). The attractiveness of FCIs stems from the fact that the bandgap Δ (which plays the same role as the Landau level spacing in the FQHE) may be set without the use of a large external magnetic field, the strength of which (~ 10 Tesla) is one of the limiting factors in the semiconductor FQHE.

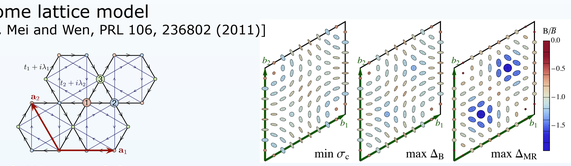
FCI phenomena constitute a non-trivial and poorly-understood generalization of the FQHE, since the Landau level Hamiltonian occupies a unique, highly symmetric point in the space of single-particle Hamiltonians. We have recently conjectured that an approach using Hilbert space geometry might prove the way for an unified theory which naturally accounts for the differences of the single particle Hamiltonians due to lattice effects.

Band geometry of fractional Chern insulators

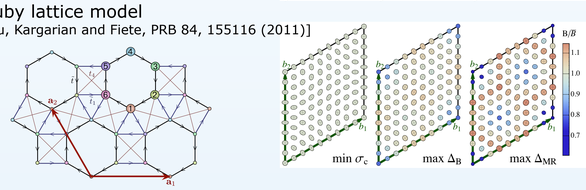
Haldane model
[F. D. M. Haldane, Phys. Rev. Lett. 61, 2015 (1988).]



Kagome lattice model
[Tang, Mei and Wen, PRL 106, 236802 (2011)]



Ruby lattice model
[Hu, Kargarian and Fiete, PRB 84, 155116 (2011)]



Plots of band geometry for couplings which minimize curvature fluctuations and maximize the many-body gap in several FCI models, respectively. Axes of ellipses are proportional to the eigenvectors of the quantum metric g(**k**). Ellipse color is given by the relative deviation of Berry curvature B(**k**) from its BZ-averaged value.

Bloch's theorem tells us that we can write eigenfunctions of a spatially periodic Hamiltonian in terms of functions u(**k**) which are periodic over the Brillouin zone. The Berry curvature is defined in terms of these as

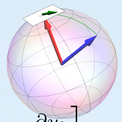
$$B(\mathbf{k}) = -i \sum_a \frac{\partial u_a^*}{\partial k_x}\frac{\partial u_a}{\partial k_y} - \frac{\partial u_a^*}{\partial k_y}\frac{\partial u_a}{\partial k_x}$$

and plays the role that the magnetic field plays in the fractional quantum Hall effect. Its integral over the Brillouin zone is a topological invariant, the first Chern number:

$$c_1 = \frac{A}{2\pi}\int d^2\mathbf{k}\, B(\mathbf{k})$$

The Hilbert space of bands admits a Fubini-Study metric

$$ds^2 = \langle\delta\psi|\delta\psi\rangle - \langle\delta\psi|\psi\rangle\langle\psi|\delta\psi\rangle$$



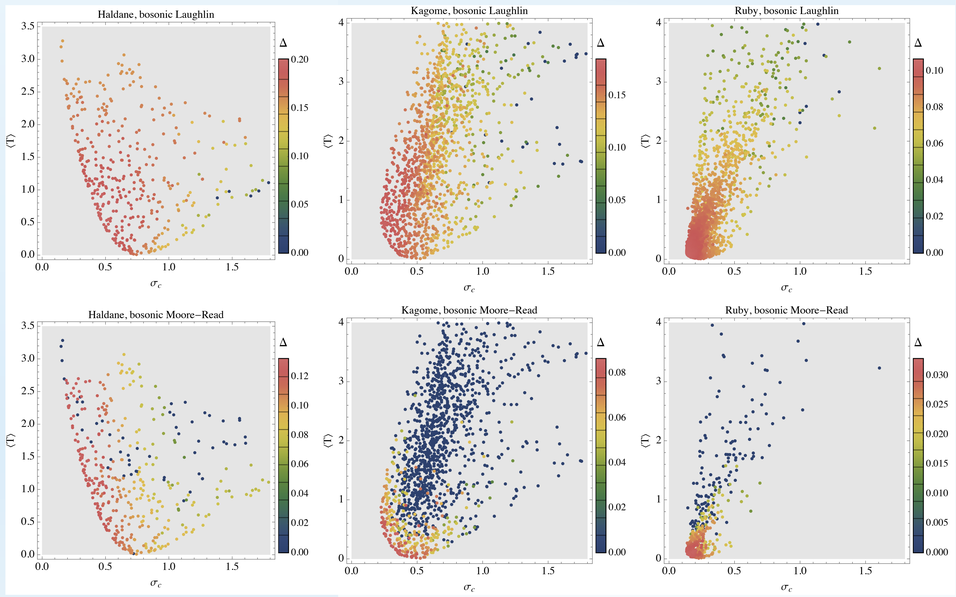
which pulls back to

$$g_{ij}(\mathbf{k}) = \frac{1}{2}\sum_a \left[\frac{\partial u_a^*}{\partial k_i}\frac{\partial u_a}{\partial k_j} - \sum_b \frac{\partial u_a^*}{\partial k_i} u_a u_b^* \frac{\partial u_b}{\partial k_j}\right] + [i \leftrightarrow j]$$

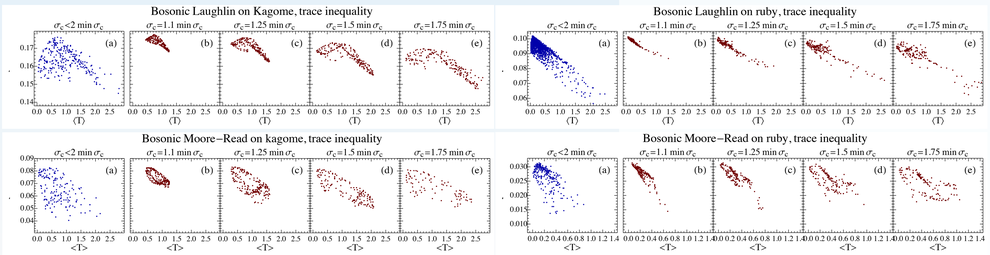
In [5] I showed that the curvature and metric are related by the following inequalities:

$$\mathrm{tr}\; g(\mathbf{k}) \geq |B(\mathbf{k})| \qquad \det\, g(\mathbf{k}) \geq \frac{1}{4}B(\mathbf{k})^2$$

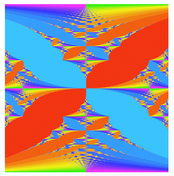
which are relevant for determining the gap to excitations of a FQHE-like state in the single mode approximation.



Numerically computed many-body gaps for FQHE-like states (dot color) in the above lattice models as a function of Berry curvature fluctuation (σ_c) and averaged metric trace inequality (<T>). Obtaining the most stable phases requires conditions on the curvature *and* the metric.



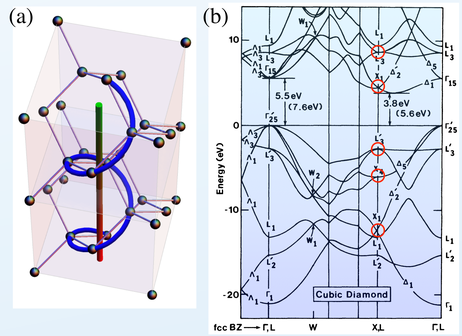
Numerically computed many-body gaps for FQHE-like states in the kagome and ruby lattice models, evaluated on isosurfaces of constant Berry curvature fluctuations in coupling space. This isolates the effect of the band metric on the gap.



A colored Hofstadter butterfly: This figure represents the phase diagram of Bloch electrons in a uniform magnetic field. The horizontal axis indicates the chemical potential and the vertical axis the flux through the system. Each color corresponds to a distinct topological phase with a particular quantized value of the Hall conductance. In [4] this model was studied perturbatively from the point of view of Landau level mixing. (Image credit: Yosi Avron)

Topological order and space group symmetries

Theorems due to Oshikawa and Hastings extending previous work of Lieb, Schultz and Mattis tell us that at fractional fillings, without spontaneous symmetry breaking, a gap can arise only in conjunction with topological order. In [7] we show that this is the case even at certain integer fillings of systems with nonsymmorphic space group symmetries.



(a) Diamond crystal structure (an example of a nonsymmorphic space group) showing one possible screw transformation (blue spiral): this is the combination of a translation and rotation around the red-green axis. (b) Band structure of diamond [data reproduced from W. Saslow, T. Bergstresser, and M. Cohen, PRL 16, 354 (1966)]. Band touchings are indicated with red circles; the Δ5 band is multiply degenerate. At the X point, these degeneracies are exact and protected by the crystal symmetry group.

Recent publications

[1] T. S. Jackson, G. Möller, R. Roy, *Geometric stability of topological lattice phases*, arXiv:1408.0843.
[2] Akshay Kumar, Rahul Roy, S. L. Sondhi, *Generalizing Quantum Hall Ferromagnetism to Fractional Chern Bands*, arXiv:1407.6000.
[3] Suk Bum Chung, Rahul Roy, *Hall conductivity in the normal and superconducting phases of the Rashba system with Zeeman field*, arXiv:1407.3883.
[4] Fenner Harper, Steven H. Simon, Rahul Roy, *Perturbative Approach to Flat Chern Bands in the Hofstadter Model*, Phys. Rev. B **90**, 075104 (2014).
[5] Rahul Roy, *Band geometry of fractional topological insulators*, Phys. Rev. B **90**, 165139 (2014).
[6] S. A. Parameswaran, R. Roy, S. L. Sondhi, *Fractional Quantum Hall Physics in Topological Flat Bands*, Comptes Rendus Physique **14**, 816 (2013).
[7] Rahul Roy, *Space group symmetries and low lying excitations of many-body systems at integer fillings*, arXiv: 1212.2944.
[8] S. A. Parameswaran, R. Roy, S. L. Sondhi, *Fractional Chern Insulators and the W-Infinity Algebra*, Phys. Rev. B **85**, 241308(R) (2012).
[9] Rahul Roy, *Topological pumps and adiabatic cycles*, arXiv:1104.1979.